Wave Digital Filters: Theory and Practice

ALFRED FETTWEIS, FELLOW, IEEE

Invited Paper

Wave digital filters (WDFs) are modeled after classical filters, preferably in lattice or ladder configurations or generalizations thereof. They have very good properties concerning coefficient accuracy requirements, dynamic range, and especially all aspects of stability under finite-arithmetic conditions. A detailed review of WDF theory is given. For this several goals are set: to offer an introduction for those not familiar with the subject, to stress practical aspects in order to serve as a guide for those wanting to design or apply WDFs, and to give insight into the broad range of aspects of WDF theory and its many relationships with other areas, especially in the signal-processing field. Correspondingly, mathematical analyses are included only if necessary for gaining essential insight, while for all details of more special nature reference is made to existing literature.

I. INTRODUCTION

The importance of digital filters is well established. A vast amount of literature exists on this subject, including many books in various languages, and digital filters are being widely used in a variety of applications. The interested reader may consult a recent tutorial paper, published in the Centennial Issue of the IEEE TRANSACTIONS ON CIRCUITS AND SYSTEMS [1], on the general topic of digital filtering.

Next to nonrecursive digital filters and digital filters composed of simple first- and second-order sections in cascade, wave digital filters (WDFs) represent a class of particular interest. They have first publicly been described in 1970 [2], [3] and a first detailed paper has been published in 1971 [4] (a condensed version thereof being [5] while [6] contains a few added results). Several early papers [4], [7]-[9] on the subject are reprinted in [10], while some other papers bearing a certain relationship to our subject are [11]-[13]. A number of books are now available that give some introductory material [14]-[28]. The purpose of the present paper is to give a brief overview of the state of the art in the field of WDFs.

WDFs represent a class of digital filters that are closely related to classical filter networks (see, e.g., [21], [29]-[38]), preferably lossless filters inserted between resistive terminations. Thus to every WDF there corresponds a reference filter from which it is derived. This relationship is the deeper reason for the many interesting properties of WDFs, but also for the somewhat more difficult access to theory and design of these digital filters. Indeed, while the translation of a given reference filter into a WDF is quite straightforward, the selection of a suitable reference structure and its design requires a certain amount of familiarity with classical filter theory, including aspects of microwave filter theory. The analogy between a WDF and its reference filter is based not on the use of voltages and currents as signal parameters, but of so-called wave quantities known from classical circuits [32], whence the name chosen for the class of digital filters considered in this paper. For sake of clarity, circuits whose topological constraints are the Kirchhoff equations (classical circuits and their multidimensional generalizations) will also be referred to as Kirchhoff circuits.

There does not essentially exist just one type of WDFs, but a whole variety of quite distinct subclasses, each of which can again be divided into various families, etc. This reflects the richness of structures available in classical circuits, and the designer thus should select that one of these reference filter structures that leads to the most appropriate overall solution for the problem at hand. Fortunately, some simple types of WDFs exist which are sufficient to meet most common requirements concerning filter performance, simplicity and modularity of structure, etc., yet such that their design can be accomplished with only very limited knowledge of classical circuits.

Several of the good properties of classical filters are a direct consequence of their passivity and, more specifically, the losslessness of the filter two-port itself. It is important, therefore, that the relationship between a WDF and a passive reference filter is preserved even after modifying the multiplier coefficients within more or less wide ranges, as is, e.g., required for deriving from the original design a bit-optimal version (i.e., a version in which the coefficients are quantized binary numbers selected according to some suitable optimality criterion) by discrete optimization. The most immediate consequences of this are that, for all major WDF structures, the excellent passband sensitivity properties of lossless filters [39]-[41] are preserved, leading to correspondingly reduced accuracy requirements for the...
filtering at the input or output. The filter has been tested successfully with actual geophysical data [403].

narrow sectors are shown in Fig. 77. If the limits of the sectors must be straight lines rather than curves as those of Fig. 77 one can achieve this to any degree of perfection, e.g., by operating the filter at higher rates, applying zero stuffing to the input signal, and providing a further low-pass filtering at the input or output. The filter has been tested successfully with actual geophysical data [403].

A quite different approach of realizing quadrant fan filters consists in cascading two 1D complex low-pass filters with cutoff frequencies at $\omega_1 = 0$ and $\omega_2 = 0$, respectively [410], or, equivalently, of applying suitable modulation to the signal before processing it in corresponding real filters [402], [403]. The filters required in this case can again be realized by means of WDFs [402], [411], but the approach appears to be less efficient than the one discussed above, especially if sectors narrower than 90° are considered.

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Part A: Papers Referenced in the Text


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**Part B: Further Papers on Wave Digital Filters and Related Subjects.**


