

Fuzzy Qualitative Trigonometry

Honghai Liu and George M. Coghill
Department of Computing Science, University of Aberdeen
AB24 3UE, Scotland United Kingdom
{hliu, ge coghill}@csd.abdn.ac.uk

Abstract - This paper proposes fuzzy qualitative representation of trigonometry (FQT) in order to bridge the gap between qualitative and quantitative representation of physical systems using Trigonometry. Fuzzy qualitative coordinates are defined by replacing a unit circle with a fuzzy qualitative circle; the Cartesian translation and orientation are replaced by their fuzzy membership functions. Trigonometric functions, rules and the extensions to triangles in Euclidean space are converted into their counterparts in fuzzy qualitative coordinates using fuzzy logic and qualitative reasoning techniques. We developed a MATLAB toolbox XTrig in terms of 4-tuple fuzzy numbers to demonstrate the characteristics of the FQT. This approach addresses a representation transformation interface to connect qualitative and quantitative descriptions of trigonometry-related systems (e.g., robotic systems).

Keywords: Qualitative reasoning, fuzzy logic, robotics

1 Introduction

Trigonometry is a branch of mathematics that deals with the relationships between the sides and angles of triangles and with the properties and application of trigonometric functions of angles. It began as the computational component of geometry in the second century BC and plays a crucial role in mathematics, engineering, etc. In order to bridge the gap between qualitative and quantitative description of physical systems, we propose a fuzzy qualitative representation of trigonometry (FQT), which provides theoretical foundations for their representation transformation of trigonometry.

It is often desirable and sometimes necessary to reason about the behaviour of a system on the basis of incomplete or sparse information. The methods of model-based technology provide a means of doing this [Kuipers, 1994]. The initial approaches to model-based technology were seminal but focused on qualitative reasoning only, providing a means whereby the global picture of how a system might behave could be generated using only the sign of the magnitude and direction of change of the system variables. This made qualitative reasoning complementary to quantitative simulation. However, qualitative and quantitative simulation forms the two ends of a spectrum; and semi-quantitative methods were developed to fill the gap. For the most part these were interval reasoners bolted on to existing qualitative reasoning systems (e.g. [Berleant, 1997]); however, one exception to this was fuzzy qualitative reasoning which integrated the strengths of approximate reasoning with those of qualitative reasoning to form a more coherent semi-quantitative approach than their predecessors [Shen and Leitch, 1993; Coghill, 1996]. Model-based technology methods have been successfully applied to a number of tasks in the process domain. However, while some effort has been expended on developing qualitative kinematic models, the results have been limited [Blackwell, 1988; Faltings, 1992] etc. The basic requirement for progressing in this domain is the development of qualitative version of the trigonometric rules. Buckley and Esami [Buckley and Esami, 2002] proposed the definition of fuzzy trigonometry from fuzzy perspective without consideration of the geometric meaning of trigonometry. Some progress has been made in this direction by Parsons [Parsons, 2001] and Liu [Liu, 1998], but as with other applications of qualitative reasoning, the flexibility gained in variable precision by integrating fuzzy and qualitative approaches is no less important in the kinematic domain. In this paper we present an extension of the rules of trigonometry to the fuzzy qualitative case, which will serve as the basis for fuzzy qualitative reasoning about the behaviour and possible diagnosis of kinematic robot devices.

1.1 Quantity Representation in Fuzzy Qualitative Reasoning

Qualitative reasoning has explored tradeoffs in representations for continuous parameters ranging in resolution from sign algebras to the hyperreals. Intervals are a well-known variable-resolution representation for numerical values, and have been heavily used in qualitative reasoning [Forbus, 1996]. A quantity space is to represent continuous values via sets of ordinal relations, it can be thought of as partial information about a set of intervals [Lee et al., 2002]. The natural mapping between quantity spaces and intervals has been exploited by a variety of systems that use intervals whose endpoints are known numerical values to refine predictions produced by purely qualitative reasoning [Kuipers, 1994]. Fuzzy intervals have also been used in fuzzy reasoning about mechatronics systems [Shen and Leitch, 1993]. Fuzzy qualitative trigonometry has chosen the concept of a quantity space due to the fact that it is semi-qualitative qualitative reasoning and the success of fuzzy qualitative simulation. In this case the quantity space is a set of overlapping fuzzy numbers, an example of which in FuSim Fuzzy Simulation [Shen and Leitch, 1993] is shown in Figure 1.

![Figure 1: A fuzzy quantity space](image)

The quantity space for every variable in the system is a finite and convex discretisation of the real number line. The
Table 1: Qualitative position description of the end-effector

<table>
<thead>
<tr>
<th>( QS_x ) (A-X)</th>
<th>( QS_y ) (A-Y)</th>
<th>( QS_z ) (A-Z)</th>
<th>( QS_{\theta} ) (A-( \theta ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>( QS_x(1) )</td>
<td>( QS_y(1) )</td>
<td>( QS_z(1) )</td>
<td>( QS_{\theta}(1) )</td>
</tr>
<tr>
<td>( QS_x(2) )</td>
<td>( QS_y(2) )</td>
<td>( QS_z(2) )</td>
<td>( QS_{\theta}(2) )</td>
</tr>
<tr>
<td>( QS_x(9) )</td>
<td>( QS_y(9) )</td>
<td>( QS_z(9) )</td>
<td>( QS_{\theta}(9) )</td>
</tr>
<tr>
<td>( QS_x(10) )</td>
<td>( QS_y(10) )</td>
<td>( QS_z(10) )</td>
<td>( QS_{\theta}(10) )</td>
</tr>
</tbody>
</table>

6 Discussions and Conclusions

A fuzzy qualitative version of traditional trigonometry has been proposed in this paper. Trigonometry in Cartesian coordinates is mapped into a fuzzy qualitative coordinate system by using fuzzy logic and qualitative reasoning techniques. FQT functions and their characteristics are derived and proved with examples through the paper. The trigonometry extension (i.e., FQT) could provide a general interface to easily communicate between the numeric world and qualitative world. Future work will focus on applying FQT to the robotics domain (e.g., qualitative kinematics and robotic communication) and process systems (e.g., reasoning about behaviours of dynamic systems).

References


