Design of 2-D FIR Digital Filters by McClellan Transformation and Least Squares Eigencontour Mapping

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Abstract—A powerful eigenfilter approach is proposed to determine the optimal coefficients of McClellan transformation. It can design arbitrary shape transformation contours to map from 1-D prototypes to 2-D FIR filters very effectively. This paper presents the design methods for 2-D fan filters with general slope and inclination angle, elliptically symmetric filters of arbitrary orientation, circularly symmetric filters, and diamond-shaped filters in details. Several numerical examples are given to demonstrate the usefulness and the efficiency of the present method.

I. INTRODUCTION

McClellan [1] has proposed a powerful technique for the design of 2-D FIR digital filters by means of 1-D to 2-D frequency transformation. It consists of mapping 1-D prototype filters into 2-D filters by a change of variables. McClellan's transformation procedure breaks the 2-D filter design problem up into two smaller problems, namely, designing the transformation mapping contours and designing the 1-D prototype. The main advantage of this technique is that it is very fast, because the computing time is devoted almost entirely to the 1-D design. Furthermore these 2-D filters have a very efficient structure for hardware implementation [2], [3].

The original McClellan transform [1] involves the substitution

\[ \cos w = F(w_1, w_2) \]

\[ = f_{00} + f_{10} \cos w_1 + f_{01} \cos w_2 + f_{11} \cos w_1 \cos w_2 \]  

(1a)

in which

\[ |F(w_1, w_2)| \leq 1. \]  

(1b)

The transform in (1) was used for the design of 2-D linear phase FIR filters, and is extended lately to

\[ \cos w = F(w_1, w_2) \]

\[ = \sum_{i=0}^{I} \sum_{j=0}^{J} t_{ij} \cos i w_1 \cos j w_2 \]

\[ + \sum_{k=1}^{K} \sum_{l=1}^{L} s_{kl} \sin k w_1 \sin l w_2 \]  

(2)

by Mersereau et al. [2] and Nguyen et al. [4] for designing quadrantly symmetric and centro-symmetric filters, respectively.

In general the coefficients of the McClellan transform are computed using optimization techniques [2], [5]. These techniques require a large computational effort. A new least square approximation technique is proposed for fast calculation of the McClellan transform coefficients. The technique we use in this paper is derived from the eigenfilter design [6], [7], [8], which has successfully been used to design 1-D and 2-D FIR filters in the optimal least square sense. It involves the integration of least square error along the desired contour; by minimizing a quadratic measure of the total error along the contour, a real eigenvector of an appropriate matrix is computed to get the McClellan transform coefficients. It is shown that this technique gives better results than those reported by the recent literature. Also the technique can put certain constraints on the transform coefficients such that the scaling of the transformation can be avoided [2], [9].

This paper is organized as follows: Section II describes the design of 2-D quadrantly symmetric fan filters with arbitrary inclination and the results are compared with those of [10]. Also we shall present the application of the eigenfilter approach for determining the McClellan transform coefficients. Sections III and IV explain the design of quadrantly elliptically symmetric filters and the elliptically symmetric filters of arbitrary orientation respectively, and then we compare the results with those of References [2], [11], and [4]. Section V discusses the design of quadrantly circularly symmetric filters and diamond-shaped filters. It is found that only one transformation coefficient needs be determined, then the solution can be easily obtained to minimize the error without the eigenfilter approach. For the results of circularly symmet-
VI. CONCLUSIONS

This paper presents a powerful eigenfilter approach to determine the optimal coefficients of McClellan transformation. The contour error minimization is extremely simple and fast through very small size matrix eigenvector computation. It can design arbitrary shape transformation contours to map from 1-D prototypes to 2-D FIR filters very effectively. Several numerical examples including 2-D fan filters, elliptically/circularly symmetric filters, and diamond-shape filters are demonstrated to show the effectiveness of this approach.

REFERENCES

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